**Discrete time integration algorithm**

# 1. **Model:**

In discrete form

・　Initial condition

・　Try to find so as to

In incremental form (which is not available when M, C, K depend upon t)

With etc.

# 2. **Solution by Newton-Raphson method**

・　**In acceleration form**

・　**In velocity form**

・　**In displacement form**

# 3. **Newmark-β method**

・ **In acceleration form**

This equation is frequently used in text books of dynamics.

・ **In velocity form**

・ **In displacement form**

In total displacement form

# 4. **HHT-α method**

Discrete model modified as

Which adopts finite difference equation of the Newmark-β method.

・ **In acceleration form**

In total acceleration form

・　**In displacement form**

# 5. **Generalized-α method**

Discrete model modified as

Which adopts finite difference equation of the Newmark-β method also. The constants of are suitably chosen so that the scheme is stable. The algorithm is unconditionally stable if the coefficients are chosen such that for ρ∞<1,

produce the Newmark method. corresponds to the HHT method. For unconstrained systems, it is second order accurate provided that

・ **In acceleration form**

・ **In displacement form**

## **5.1 Modified Form** (M.Arnold, O. Bruls: Convergence of the generalized-α scheme for constrained mechanical systems, Multibody Systems Dynamics, 18(2), 185-202, 2007)

An acceleration-like variable **a** in defined by the recurrence relation as:

The generalized scheme is obtained using **a** in the Newmark integration formula:

In this formula, we need remember both and .

**・ In acceleration form**:

・ **In displacement form**

# 6 Central-difference

・ **In displacement form**

In total displacement form

　In incremental form

・　**Forward increment Lagrange multiplier method** (N.J.Carpenter, R.L.Taylor, M.G. Katona: Lagrange constraints for transient finite element surface contact, Int. J. Num. Meth. Eng., 32(1991), 103-128)

With

Step 1: Calculate

Step 2: Calculate

Step 3: Calculate

Step 4: and update

Gauss-Seidel iteration of step 2

Step 2.1: Initialize

Step 2.2: Calculate Fc->uc->ucg

Step 2.3:

Step 2.4:

Step 2.5: Convergence test

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4

5

λ１

λ2

Or briefly (I=1,2,…ncontact)